Uncertainty Analysis in Lorentz Force Eddy Current Testing

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The present paper addresses the analysis of uncertainties in the framework of the nondestructive evaluation technique Lorentz force eddy current testing. A generalized polynomial chaos expansion is used in order to quantify the impact of multiple unknown input parameters. In this context, the velocity and the conductivity of the specimen as well as the magnetic remanence and the lift-off distance of the permanent magnet are modelled as uniform distributed random variables. A comparison to experimental results show good agreement to numerical predictions. A sensitivity analysis by means of a sobol decomposition revealed that the magnetic remanence and the lift-off distance contribute to more than 80% to the total variance of the resulting Lorentz force profile.

Index Terms**—Eddy currents, Finite element analysis, Nondestructive testing, Uncertainty.**

I. Introduction

The analysis of uncertainties plays an important role during the design process of new systems, especially in the framework of nondestructive testing (NDT) [1], [2]. Lorentz force eddy current testing (LET) is an NDT technique to evaluate electrically conductive materials. The general principle is illustrated in Fig. 1. It is based on the induction of eddy currents due to relative motion between a permanent magnet and the object under test [3]. The resulting Lorentz force acting on the magnet is measured and used to analyse the quality of the specimen. By means of numerical simulations, it is possible to predict the Lorentz force profile [4]. However, in this context the intrinsic variability of the input parameters were yet not accounted. Hence, one can not rely on a single deterministic simulation and a quantification of uncertain model data on the output is essential. As a result, it is possible to identify and reduce prior sources of uncertainty in order to improve the experimental setup.

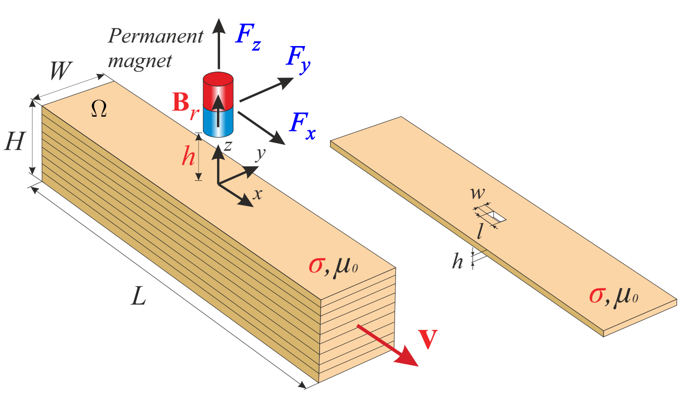

Fig. 1. The principle of LET. The red parameters are modelled as uniform random variables and the blue values indicate the output quantities (adopted from [5]).

II. Methods

A. Numerical analysis

In the present framework, the velocity v, the electrical conductivity σ, the lift-off distance h and the magnetic remanence B_r are modelled as uniform distributed random variables. The associated limits are derived by our experimental setup (Table I). The quantities of interest are the drag- and lift-component of the Lorentz force F_i with $i \in \{x, z\}$. A non-intrusive generalized polynomial chaos expansion (gPC) is applied in order to investigate the propagation of uncertainties throughout the system under investigation [6]. Therefore, the magnetic convection diffusion equation is solved in its quasi-static form assuming low to moderate magnetic Reynolds numbers $R_m = \mu_0 \sigma v W/2 \leq 1$:

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{A} - \mathbf{M} \right) = \sigma \left(-\nabla\varphi + \mathbf{v} \times \nabla \times \mathbf{A} \right) \quad (1)$$

$$\nabla \cdot \left[\sigma \left(-\nabla\varphi + \mathbf{v} \times \nabla \times \mathbf{A} \right) \right] = 0. \quad (2)$$

The magnetic field is expressed by means of the magnetic vector potential $\mathbf{B} = \nabla \times \mathbf{A}$ using a Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ and the electric field is modelled with a scalar electric potential formulation $\mathbf{E} = -\nabla\varphi + \mathbf{v} \times \nabla \times \mathbf{A}$. The remanence of the permanent magnet is included in the magnetization vector $\mathbf{M} = \mathbf{B}_r/\mu_0 = \mathbf{B}/\mu_0 - \mathbf{H}$. The Lorentz force on the magnet $\mathbf{F} = -\int_\Omega \mathbf{J} \times \mathbf{B}\, \mathrm{d}\Omega$ is expressed by means of basis functions $\psi_k(\boldsymbol{\xi})$ which are build up by Legendre polynomials:

$$F_i(\boldsymbol{r}, \boldsymbol{\xi}) = \sum_{k=0}^{\infty} \hat{u}^{(k)}(\boldsymbol{r})\, \psi^{(k)}(\boldsymbol{\xi}) \approx \sum_{k=0}^{N_c - 1} \hat{u}^{(k)}(\boldsymbol{r})\, \psi^{(k)}(\boldsymbol{\xi}). \quad (3)$$

The random variables are summarized in the vector $\boldsymbol{\xi}$ and the argument $\boldsymbol{r}$ indicates the location of the magnet. Each position between the magnet and the specimen is treated independently. The regression method [7] is used to determine the coefficients $\hat{u}^{(k)}(\boldsymbol{r})$ of N_r different magnet positions constituted in the matrix $[\mathbf{U}]$ of size $[N_c \times N_r]$:

$$[\Psi]\, [\mathbf{U}] = [\mathbf{S}], \quad (4)$$

where $[\Psi]$ denotes the gPC-matrix of size $[N_g \times N_c]$. The solutions are gathered in the $[N_g \times N_r]$ matrix $[\mathbf{S}]$ obtained by N_g deterministic simulations. The number of forward

TABLE I
BOUNDS OF UNIFORM DISTRIBUTED RANDOM VARIABLES.

v in m/s	σ in MS/m	h in mm	B_r in T
[0.495...0.505]	[30.35...30.97]	[0.9...1.1]	[1.195...1.235]

simulations depends on the resolution of the chosen grid in the random space.

The Sobol coefficients quantify the influence of the individual random variables $\boldsymbol{\xi}$ on the total variance σ. They are determined by introducing a set $\mathcal{A}_i$ of multiindices $\boldsymbol{\alpha} = (\alpha_1...\alpha_N)$. The set $\mathcal{A}_{i_1,...,i_{N_s}}$ contains all $\boldsymbol{\alpha}$ pointing to an orthogonal polynomial $\psi(\boldsymbol{\xi})$ which depends on the variable(s) corresponding to the respective Sobol coefficient $S^{(\sigma)}_{i_1,...,i_{N_s}}$. Hence, the Sobol coefficients are given by:

$$S^{(\sigma)}_{i_1,...,i_{N_s}}(\boldsymbol{r}) = \frac{1}{\sigma^2} \sum_{\boldsymbol{\alpha}\in\mathcal{A}_{i_1,...,i_{N_s}}} \left(\hat{u}^{(\boldsymbol{\alpha})}(\boldsymbol{r})\right)^2 \int_{-1}^{+1} \left(\psi^{(\boldsymbol{\alpha})}(\boldsymbol{\xi})\right)^2 \mathrm{d}\boldsymbol{\xi}. \tag{5}$$

The number of Sobol coefficients N_s depends on the number of input random variables N and is $N_s = 2^N - 1 = 15$.

B. Experimental setup

In the setup under investigation, a cylindrical permanent magnet with a diameter of 15 mm and a height of 25 mm is used. The specimen is made of stacked aluminium sheets separated by thin paper sheets. It has a total dimension of $[L, W, H] = [250 \times 50 \times 50]$ mm. Each sheet has a thickness of 2 mm. As an admissible approximation, an anisotropic conductivity profile is assumed ($\sigma_{xx} = \sigma_{yy}$, $\sigma_{zz} = 0$). One of the sheets contains a defect of size $[l, w, h] = [12 \times 2 \times 2]$ mm which is located in the lateral center of the specimen at a depth of $d = 2$ mm. In the same way, the magnet is also located in the lateral center such that the specimen is analysed in its centerline. The remaining parameters are chosen such that they correspond to the mean values given in Table I.

III. RESULTS

The gPC is performed at $N_r = 11$ relative positions between the permanent magnet and the specimen. In the present case, it was sufficient to expand the gPC until the order $p = 3$. Considering a maximum order expansion, this yields together with $N = 4$ random variables in $N_c = \binom{N+p}{N} = 35$ coefficients. The random space is sampled by a Gauss-Legendre tensored grid of $m = 3$ points in every dimension ($N_g = m^N$), which results in a total number of $N_r N_g = 891$ deterministic quasi-static FEM simulations. Each calculation takes ~ 10 s leading to a total simulation time of about $2 - 3$ h. A typical deterministic Lorentz force profile obtained with a single run across the specimen with the corresponding mean values from Table I is shown in Fig. 2. The plot shows the drag-force F_x and the lift-force F_z obtained by numerical simulations and experiments. The side-force F_y vanishes for symmetry reasons. Moreover, the graph shows the resulting uncertainty intervals $\mu_{x,z} \pm 3\sigma_{x,z}$ obtained by the gPC. It can be observed that the measurements are in the predicted range when uncertainties of the defined input parameters are taken into account. The

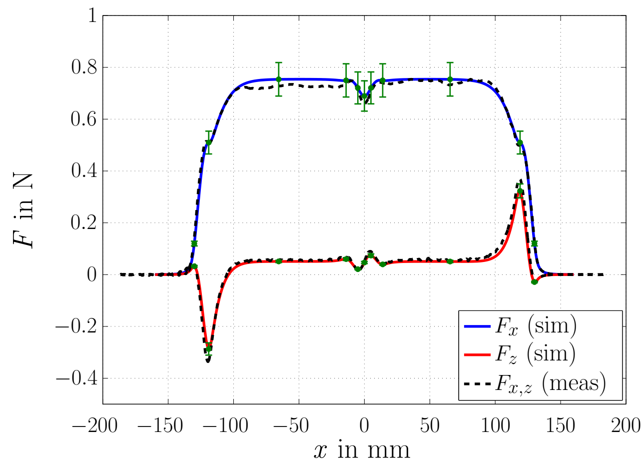

Fig. 2. Numerical simulations and experimental results of the Lorentz force profile shown together with the associated uncertainty intervals $\mu_{x,z} \pm 3\sigma_{x,z}$.

Sobol coefficients are determined at each individual position and averaged with respect to the x-position of the magnet. The corresponding values for F_x and F_z are shown in Table II. It is observed that the first four (linear) Sobol coefficients are significant and cover 99.9% of the total variance.

TABLE II
AVERAGED LINEAR SOBOL COEFFICIENTS OF F_x AND F_z.

S_x in %	S_B	S_h	S_σ	S_v
F_x / F_z	46.3 / 40.3	45.2 / 35.5	4.2 / 12.3	4.2 / 11.8

IV. CONCLUSION

The present study shows that the analysis of uncertainties by means of gPC based methods can be readily used for extended experimental validations in the framework of LET. A Sobol decomposition revealed that the magnetic remanence and the lift-off distance have the greatest influence. Hence, in order to reduce the variance of the resulting Lorentz force, it is desirable to reduce the uncertainty of these parameters first.

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